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1987

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citation for published version (APA)

Palm, F. C., & Winder, C. C. A. (1987). *The life cycle consumption model under structural changes in income and moving planning horizons*. (Serie Research Memoranda; No. 1987-17). Faculty of Economics and Business Administration, Vrije Universiteit Amsterdam.

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THE LIFE CYCLE CONSUMPTION MODEL UNDER
STRUCTURAL CHANGES IN INCOME AND MOVING
PLANNING HORIZONS

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Researchmemorandum 1987-17

maart '87



**VRIJE UNIVERSITEIT
FACULTEIT DER ECONOMISCHE WETENSCHAPPEN
AMSTERDAM**

1997

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THE LIFE CYCLE CONSUMPTION MODEL

UNDER STRUCTURAL CHANGES IN INCOME AND MOVING PLANNING HORIZONS

Franz C. Palm* and Carlo C. A. Winder**

March 1987

ABSTRACT

The purpose of the paper is to investigate intertemporal decision-making by the consumer. We start the analysis with a discussion of the life cycle hypothesis, cast into a framework similar to that proposed by Hall (1978). We explicitly link the stochastic model for consumption to the characteristics of the income process. In the empirical part we estimate the model for quarterly data on consumption for the Netherlands. We pay attention to the implications of a structural shift in the income process for the consumption model and we argue that the life cycle hypothesis can only account for the data when we assume a structural change in one of the parameters.

As an alternative we extend the model by introducing a moving planning horizon. The resulting model describes the data fairly well without requiring an ad-hoc assumption on a structural change in the parameters of the life cycle model. Moreover it leads to a specification with an (error) correction term as proposed by Davidson, Hendry, Srba and Yeo (1978).

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1 Introduction.

Since Modigliani and Brumberg (1955) put forward the life cycle consumption hypothesis, this theory has been extensively analysed and tested, using both cross-section and time series data. Their work has kept a prominent position among economic theories of consumption. Among the many articles that deal with extensions and refinements of the life cycle theory, an important contribution is due to Hall (1978). He formulates the life cycle hypothesis as an intertemporal decision problem under uncertainty and shows that the first order conditions for an intertemporal optimum have straightforward implications for the serial correlation properties of the time series data on consumption. More specifically, the marginal utility of consumption is shown to be generated by a first order autoregressive process. Many authors have pursued Hall's approach, see e.g. Bilson (1980), Flavin (1981), Muellbauer (1983), Wickens and Molana (1983) and for a survey Deaton (1985).

Under the assumption that income is exogenous, the stochastic process of consumption is just a transformation, accomplished by the mathematical model, of the stochastic properties of income. The analogy with physical experiments is obvious. Income is the input variable and consumption is the output variable. To put it differently the life cycle theory generates a number of restrictions between the process for consumption and income. Unanticipated structural changes in the income process have for instance a specific effect on consumption. One of the purposes of the paper is to discuss the relationship between consumption and income and to test the implications of these relationships. The extension with respect to Hall's approach is obvious. The stochastic properties of consumption are analyzed in the light of those of income. In the second part, we look at the relationship between consumption and income when the consumer maximizes the intertemporal expected utility of consumption but shifts the planning horizon further ahead in the future as time goes on. This model leads to a relationship between income and consumption which is highly comparable with the mechanism underlying the consumption function proposed by Davidson et al. (1978). In particular we find as an explaining variable the error

correction term of their consumption function. Throughout the paper we make the assumption of rational expectations, that is, we assume that the subjective distribution of the income process used in the utility maximization problem coincides with the actual distribution of income.

The paper is organized as follows. In section 2 we analyze the life cycle hypothesis for the utility function with constant absolute risk aversion. The framework is similar to that of Hall. The main difference is that he assumes that the consumer takes into account the complete distribution of labor income, whereas we assume that he uses only the information on expected future labor income. Under both regimes the model leads to a completely specified stochastic process for consumption. The empirical analysis shows that the model provides a good description of the data, given we are prepared to extend the model to account for a structural break.

In section 3 we assume that the consumer adjusts the planning horizon as time goes on. The resulting model removes the need to postulate a structural change in the parameters. It takes the form of an error correction model. We will argue that in our framework it is more appropriate to speak about a correction term, because no error is involved. From the empirical part of this section, we conclude that the theory is in accordance with the data. The model is examined more deeply in section 4. The first part of the section is devoted to a comparison with Hall's model. Moreover it is shown that the model is observationally equivalent to a model analyzed in Palm and Winder (1987). Finally, section 5 concludes the study.

2 The life cycle model.

2.1 Theory.

In this section we discuss the life cycle model. We assume a time additive Von Neumann Morgenstern utility function. At each time period t the consumer solves the following nonstochastic utility maximization problem

$$\text{MAX} \sum_{i=0}^{T-t} \beta^i U(c_{t+i}) \quad (2.1)$$

$$\text{s.t.} \sum_{i=0}^{T-t} (1+r)^{-i} c_{t+i} = (1+r)a_{t-1} + y_t + \sum_{i=1}^{T-t} (1+r)^{-i} E(y_{t+i} | I_t)$$

with $U' > 0$, $U'' < 0$, where U' and U'' are the first and second derivatives of U with respect to c respectively. Real consumption and real labor income is denoted by c_{t+i} and y_{t+i} respectively, a_{t-1} is cumulated real financial wealth, T denotes the life time, β is the time preference parameter, $0 < \beta < 1$, and r is the real interest rate, which is assumed to be constant ($0 < r < 1$). E denotes the familiar expectation operator, and I_t is the information set available at time t and used by the consumer. We assume that the relevant information consists of past realizations of income or consumption. Because of the correspondence stressed in the introduction, we may concentrate on either the past of income or the past of consumption without changing the nature of the information set.

To arrive at an operational model it is necessary to choose a specific functional form for U . In this paper we study the utility function with constant absolute risk aversion

$$U(c) = -\gamma^{-1} \exp(-\gamma c) \quad , \quad \gamma > 0 \quad (2.2)$$

The assumptions underlying the model (2.1) differ from those often made when consumers are assumed to maximize the expected present value of the utility of present and future consumption given the life time budget

constraint. In section 4, we shall compare the two models and show that for the utility function (2.2) and no structural change in the income process the two models are observationally equivalent.

The first order conditions implied by (2.1) and (2.2) are

$$c_{t+i}^t = i\gamma^{-1}\ln[\beta(1+r)] + c_t^t, i = 1, \dots, T-t, \quad (2.3)$$

where c_{t+i}^t denotes the consumption plan for period $t+i$ made at time t . For period t , we have $c_t^t = c_t$ as the realization. After substitution of (2.3) into the intertemporal budget constraint, we get for c_t

$$\eta_{T-t}c_t^t + \gamma^{-1}\ln[\beta(1+r)]\tau_{T-t} = (1+r)a_{t-1} + y_t + \sum_{i=1}^{T-t} (1+r)^{-i}E(y_{t+i}|I_t), \quad (2.4)$$

where

$$\eta_k = \sum_{i=0}^k (1+r)^{-i} \quad \text{and} \quad \tau_k = \sum_{i=1}^k i(1+r)^{-i}.$$

The parameters of the exact relationship (2.4) could be estimated provided the first moments of income are given. Moreover to estimate (2.4) a disturbance term has to be introduced. Notice also that Friedman's (1957) Permanent Income Hypothesis and Modigliani & Brumberg's Life Cycle Hypothesis arise as a special case of (2.4), when the "constant" term on the left hand side equals zero. To investigate the dynamics in the consumption series, it is convenient to relate c_t to c_{t+1} . For period $t+1$ the corresponding formula for consumption will be

$$\eta_{T-t-1}c_{t+1}^{t+1} + \gamma^{-1}\ln[\beta(1+r)]\tau_{T-t-1} = (1+r)a_t + y_{t+1} + \sum_{i=1}^{T-t-1} (1+r)^{-i}E(y_{t+1+i}|I_{t+1}). \quad (2.5)$$

Dividing (2.5) by $1+r$, substituting $a_t = (1+r)a_{t-1} + y_t - c_t$, and subtracting (2.3) leads to

$$c_{t+1}^t - c_t^t = \gamma^{-1} \ln[\beta(1+r)] + \eta_{T-t-1}^{-1} \left\{ y_{t+1} - E(y_{t+1} | I_t) + \sum_{i=1}^{T-t-1} (1+r)^{-i} [E(y_{t+i+1} | I_{t+1}) - E(y_{t+i+1} | I_t)] \right\} . \quad (2.6)$$

An advantage of this procedure is that we have eliminated financial wealth. Because of the scarcity of reliable data on this variable (see Modigliani (1975)), we hope that concentrating on (2.6) will lead to more trustworthy conclusions on the life cycle model.

When we specify the process for labor income, the model for consumption is completely specified. Let us assume that the change in income is generated by a stationary process with moving average representation

$$y_{t+1} = y_t + \delta + \sum_{i=0}^{\infty} \psi_i \nu_{t+1-i}, \quad \psi_0 = 1, \quad \sum_{i=0}^{\infty} \psi_i^2 < \infty, \quad \sigma^2(\nu_t) = \sigma_\nu^2, \quad (2.7)$$

which is operative both in periods t and $t+1$. As the moments of y_{t+i} , conditionally on some initial value, satisfy

$$E(y_{t+i} | I_{t+1}) - E(y_{t+i} | I_t) = (\psi_0 + \dots + \psi_{i-1}) \nu_{t+1}, \quad i=1, \dots, T-t, \quad (2.8)$$

we get after substituting (2.8) into (2.6)

$$c_{t+1}^t - c_t^t = \gamma^{-1} \ln[\beta(1+r)] + \eta_{T-t-1}^{-1} \left\{ \sum_{i=0}^{T-t-1} (1+r)^{-i} (\psi_0 + \dots + \psi_i) \right\} \nu_{t+1}. \quad (2.9)$$

When we define the consumption innovation $\varepsilon_{t+1} = c_{t+1} - E(c_{t+1} | I_t)$ we find

$$\varepsilon_{t+1} = \eta_{T-t-1}^{-1} \left\{ \sum_{i=0}^{T-t-1} (1+r)^{-i} (\psi_0 + \dots + \psi_i) \right\} \nu_{t+1}. \quad (2.10)$$

Hence, the consumption innovation is a linear transformation of the income innovation and its variance is given by

$$\sigma^2(\varepsilon_{t+1}) = \eta_{T-t-1}^{-2} \left\{ \sum_{i=0}^{T-t-1} (1+r)^{-i} (\psi_0 + \dots + \psi_i) \right\}^2 \sigma_\nu^2. \quad (2.11)$$

Equation (2.9) can also be estimated, but unlike (2.4) there is a disturbance term in (2.9). Relationship (2.10) can be used to relate ε_{t+1}

to income. This illustrates the statement in the introduction that given the income process the stochastic process for consumption is completely specified.

Notice that the variance of the consumption innovation in (2.11) is age/time-dependent. When the model (2.9) has to be estimated from aggregate real per capita data, it is not sufficient to assume that these data correspond to a representative consumer. When the age structure of the population and the income distribution over different age groups are fairly stable over time, the assumption of a constant variance for aggregate real per capita consumption is expected to be appropriate.

When an unexpected structural change occurs in the income process, the effect on consumption can be traced by using expression (2.6). An unanticipated change in δ in (2.7) leads for instance to a step change in the consumption level, which, because of the assumption of rational expectations, is completed as soon as the structural shift in income arises.

As the constancy of $\sigma^2(\nu_{t+1})$ in (2.7) is not required for deriving (2.9) and (2.10), we see that any heteroscedasticity of the income innovations should be reflected in the consumption series. We can for instance introduce an autoregressive conditional heteroscedasticity (ARCH) process (see Engle(1982)) for ν_{t+1} , and we can generalize (2.7) by assuming that y_t is generated by an ARIMA-process with innovations being ARCH (see Weiss(1984)). When ν_t in (2.7) follows an ARCH process of order p , then because of (2.10) ε_t should follow an ARCH process of the same order. A nice feature of ARCH models is that they can handle the clusters of outliers. This feature makes ARCH-processes of great potential interest. When we are prepared to relax the assumption of fully rational expectations, we may find consumption innovations that can be modeled as an ARCH process, even in case of absence of heteroscedasticity of the ARCH-type in the income process. We have seen that a structural change in the income series leads to a step change in the consumption level. When the consumer incorrectly incorporates a shift in the income process in his decision, he will become aware of this after a while, and adjust his consumption level accordingly (with a small correction for his error). This will lead to a new step change, but now in the opposite direction. ARCH-processes can probably be used to model this kind of behavior. Of

course we can try to build more sophisticated models that allow for gradual learning by the consumer. This will probably lead to complicated models. Besides that, any specific choice for a learning scheme may be arbitrary, so that using ARCH processes seems to be fruitful and will catch the essential features of the consumption series. Therefore, even in case of homoscedasticity of the income process, it is plausible that ARCH structures occur in the consumption series. However, when we stick to the assumption of rational expectations there exists a 1-1 correspondence between the stochastic properties of both series.

So, to evaluate the theoretical model (2.1) we can analyze the random walk specification (2.9). In addition, a number of criteria, originating from the fact that the stochastic behavior of the output variable, viz. consumption, is a one to one transformation established by (2.1), of the stochastic process of the input variable, viz. income. The relationship between the consumption and the income process yields additional restrictions to test the life cycle theory. This will be carried out in the next subsection.

2.2 Empirical results.

In this subsection our concern will be to test the implications of the theoretical model (2.1) using quarterly data of the Netherlands. Quarterly data on labor income for 1968(1)-1984(4), and on total consumption for the period 1967(1)-1984(4) have been kindly provided by the Centraal Planbureau. The nominal series have been deflated by the price index of total consumption and they have been divided by the size of the population to obtain per capita series. The base year is 1980. The data used are given in figures 1 and 2. As the appropriate notion in the life cycle theory is consumption rather than consumption expenditure, we have also estimated the model with data on nondurable consumption per capita only. This series has been constructed by multiplying total consumption by the nondurable consumption shares. A short description is given in appendix A. The results for real nondurable consumption per capita are given in appendix B.

As the stochastic behavior of consumption is implied by the stochastic

Fig. 1 Real labor income per capita in the Netherlands, 1968(1)-1984(4).

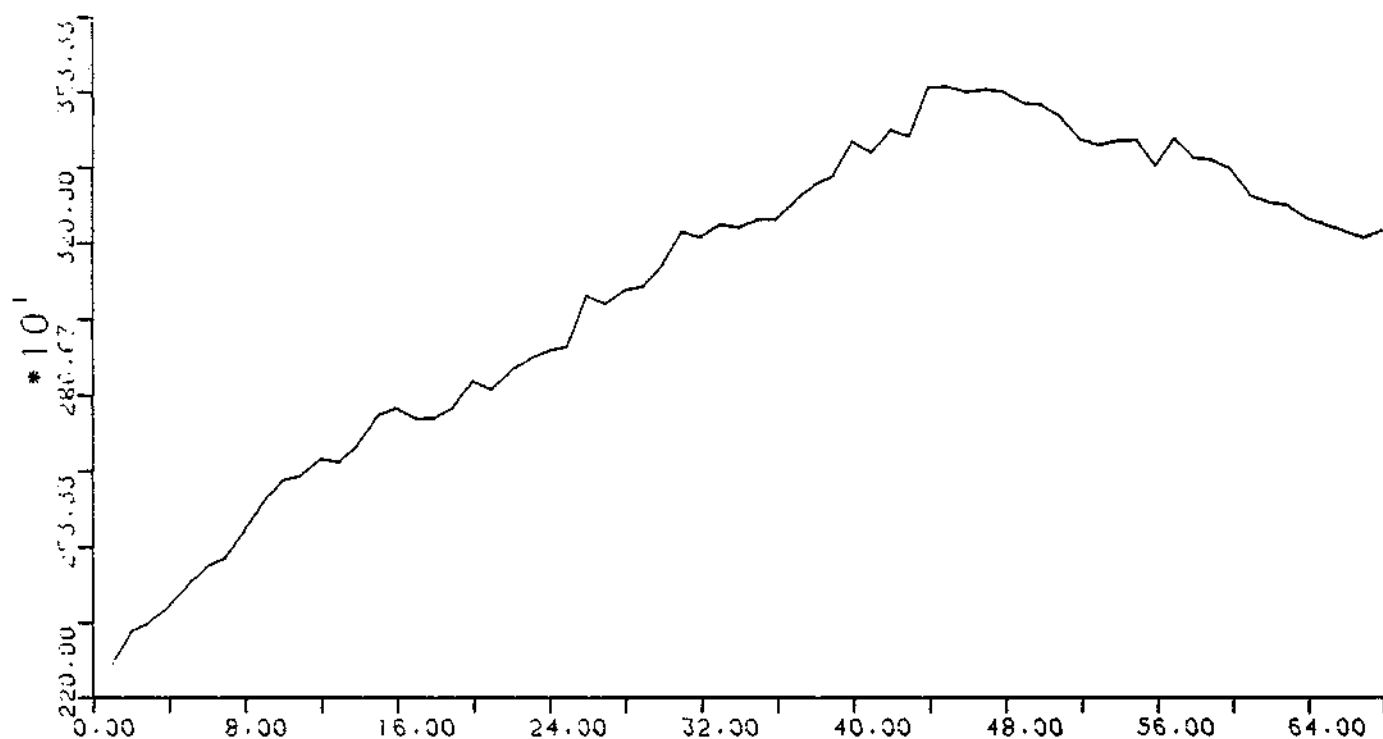
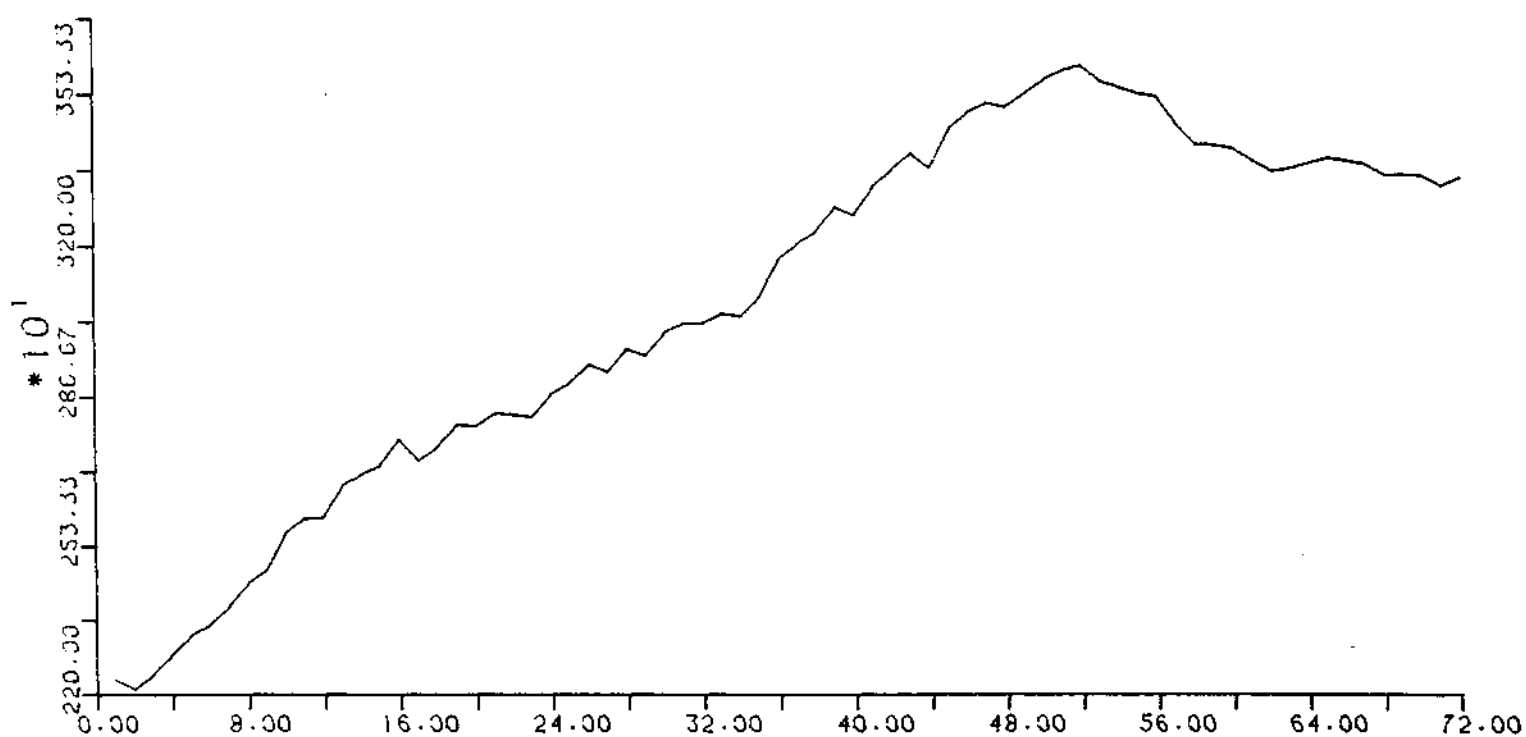


Fig. 2 Real total consumption per capita in the Netherlands, 1967(1)-1984(4).



process of income, it is natural to start by examining the income process. Inspection of figure 1 clearly shows that the change in income is not stationary. We have divided the sample period in three subperiods 1968(2)-1970(4), 1971(1)-1978(4) and 1979(1)-1984(4) respectively and calculated the autocorrelation function (ACF). For the second and third subperiod only the first order autocorrelation is significantly different from zero. In particular the values are -.41 and -.38 respectively. For the first subperiod none of the autocorrelations and partial autocorrelations is significantly different from zero. As the number of observations is only 11, this result may not be surprising. Therefore we decide to fit a MA(1)-process for Δy_t for the whole sample period, with a shifting constant. Estimation by the conditional maximum likelihood (ML)-method yields

$$\begin{aligned} \Delta y_t = & 40.46d_{1t} + 25.19d_{2t} - 13.01d_{3t} + \nu_t - .428\nu_{t-1} \\ & (7.81) \quad (8.56) \quad (3.81) \quad (3.72) \end{aligned} \quad (2.12)$$

$$t(63) = 2.524 \quad ; \quad \sigma_\nu^2 = 809.6$$

where d_{it} is a dummy variable having the value 1 in subperiod i , and 0 otherwise and t -ratio's are reported between parentheses. The value of the t -statistic for the hypothesis that the coefficients in the first two subperiods are equal, denoted by $t(63)$ is significant. Inspection of the residuals does not show any significant correlation. We find three outliers for 1974(2), 1978(4) and 1982(1). The Box-Pierce (BP) and the Ljung-Box (LB) test statistic based on s residual autocorrelations, have been computed for $s=4, 8, 12$ and 16. The results can be found in Table 1. They are not significant at commonly used significance levels. Next, we consider the constancy of the variance of the disturbance term. We have carried out a Lagrange Multiplier (LM) test for the null hypothesis that ν_t in (2.12) has a constant variance against the alternative hypothesis that the disturbance ν_t has an ARCH-structure

$$\sigma^2(\nu_{t+1} | I_t) = \alpha_0 + \sum_{i=1}^p \alpha_i \nu_{t-i}^2$$

The results are reported in Table 1 for $p=1$ and $p=4$ as $\eta(1)$ and $\eta(4)$ respectively. Clearly the test of an ARCH-structure for the income series is not significant. Finally, we check the normality of the income series using the test suggested by Lomnicki (1961). When we define

$$m_j = T^{-1} \sum_{t=1}^T \nu_t^j, j=2,3,4 \quad \text{and} \quad G_1 = m_3 m_2^{-3/2}, G_2 = m_4 m_2^{-2} - 3,$$

then Lomnicki proved that if ν_t is Gaussian and stationary, for large T , both G_1 and G_2 are normally distributed with zero means and variances that depend on the autocorrelations of ν_t . The values of the statistics $S_1 = G_1 / \sqrt{\text{var}G_1}$ and $S_2 = G_2 / \sqrt{\text{var}G_2}$, based on the first 36 autocorrelations are given in Table 1. They are highly insignificant, and do not lead to rejection of normality.

TABLE 1 Test statistics for model (2.12)

P	BP	LB
4	1.03	1.12
8	2.98	3.40
12	5.38	6.37
16	5.66	6.75
$\eta(1)$.15	
$\eta(4)$	3.22	
S_1	.26	
S_2	.07	

From the results in Table 1 we conclude that the specification (2.12), with the normality assumption of ν_t , provides a fairly good description of the income process.

Inspection of figure 2 immediately reveals that the consumption series is not stationary. In particular, the slope of consumption becomes negative at the end of the 1970's. This is not in accordance with the theoretical model. As the drift parameter of the random walk process for consumption (2.9) depends on parameters that characterize consumer behavior only, this

change of the sign can only be explained within the theoretical framework by a change in the parameters of the decision problem (2.1). It is not unrealistic to assume that the time-preference parameter β has changed as a result of the increased uncertainty about the future. Events such as the second oil crisis and a policy change aiming at a drastic reduction of public budget deficits can have had an impact on the time preference of the consumers. The consequences of a decrease of β to β^* can be traced by using the appropriate expressions of subsection 2.1. They will lead to a persistent downward adjustment of the drift parameter of (2.9), which is only completed after two periods. For the first period we find a positive step change of the drift parameter in (2.9) of the order $\eta_{T-t-1}\tau_{T-t-1}\gamma^{-1}\ln[\beta\beta^{*-1}]$. To understand why the increased uncertainty about income leads first of all to a positive step change followed by a negative one, we have to bear in mind that the lower appreciation of future consumption has two opposite effects. To keep future planned consumption at the current level, the consumer will have to increase his savings. This will be at the expense of present consumption. Because the consumer is risk averse ($\gamma > 0$), he will act on this trade-off by choosing for the certainty of actual consumption instead of deferring consumption to the future. The distribution of his life time wealth over the different periods will be adjusted to the benefit of present consumption.

In subsection 2.1 it was shown that the change in the constant term of the income process will give rise to a step change in the consumption model. Let us assume that the constant term δ moves to δ^* . Using the formula (2.6), it is easy to show that the step change will be equal to $(\delta^* - \delta)[1 + \eta_{T-t-1}\tau_{T-t-1}]$. Therefore, both in 1971(1) and 1979(1) we should expect a negative adjustment in the consumption level.

In a tentative analysis we investigate the correlation structure of the consumption series over different subperiods. In particular, the ACF and the PACF for the periods 1967(2)-1970(4), 1971(1)-1979(4) and 1980(1)-1984(4) do not suggest that the random walk specification has to be rejected. Therefore we conclude that the correlation structure of consumption is fairly well in agreement with the theoretical model. Let us examine the model in more detail. The following equation is in accordance with the life cycle theory

$$\Delta c_t = 28.61d_{1t} - 12.45d_{2t} - 76.86d_{3t} + 1.84d_{4t} - 17.29d_{5t} \quad , \quad (2.13)$$

$$\sigma^2(\varepsilon_t) = 703.68$$

where $d_{1t}=1$ for 1967(2)-1979(4)

$d_{2t}=1$ for 1980(1)-1984(4)

$d_{3t}=1$ for 1971(1)

$d_{4t}=1$ for 1979(1)

$d_{5t}=1$ for 1979(4).

The dummy variables d_{3t} and d_{4t} are included as a result of the structural changes in the income process whereas d_{2t} and d_{5t} emerge because of the presumed change in the time preference parameter at the turning point in the consumption series.

The residuals do not exhibit any correlation. For the residual ACF only r_{16} takes a significant value. We find two significant residuals for 1977(4) and 1978(1). In Table 2, we give the values of the BP and LB test-statistic, based on the first 4, 8, 12 and 16 residual autocorrelations. They are not significant. Notice that the sharp increase when we pass from 12 to 16 is heavily influenced by the large value of r_{16} . To check whether the slope of the consumption line is constant during the period 1967(2)-1979(4), we have also estimated the model with two separate slope coefficients α_1 and α_2 for the subperiods 1967(2)-1970(4) and 1971(1)-1979(4) respectively. The results are $\hat{\alpha}_1=36.00$ (5.28) and $\hat{\alpha}_2=25.25$ (5.50). A t-test of the equality of α_1 and α_2 yields an insignificant value: $t(65)=1.309$.

Above we found that the normality and homoscedasticity for Δy_t do not have to be rejected. Given that income is normally distributed and homoscedastic, the theory predicts that consumption should follow a normally distributed, and homoscedastic random walk process. In Table 2 we report the test-statistics for the ARCH structure and the normality of ε_t respectively. Both test are insignificant, so that we conclude that with respect to these arguments the empirical results are in accordance with the theory.

TABLE 2 Test statistics for model (2.13)

p	BP	LB
4	3.05	3.20
8	5.28	5.76
12	9.81	11.29
16	18.00	22.12
$\eta(1)$.43	
$\eta(4)$	3.36	
S_1	-.11	
S_2	.003	

Next, we consider the point estimates. Using expression (2.10) we find for the consumption innovation

$$\varepsilon_t = (1 - \theta + \theta\eta_{T-t-1}^{-1}) \nu_t, \quad (2.14)$$

where θ is the MA-parameter of (2.12). As $\hat{\theta} = .428$, we have as an implication of the theoretical model that the variance of the consumption innovation is smaller than that of the income innovation. A comparison of the values reported in (2.12) and (2.13) confirms the theory on this point. For the appraisal of the step changes, we have to keep in mind that the coefficients of d_{3t} , d_{4t} and d_{5t} absorb the joint effect of the adjustment in the consumption level and the transformed income innovation. From (2.12) we have an estimate of the income innovation and the MA-parameter. With this knowledge we can show that the coefficients of d_{3t} and $-d_{5t}$ should be negative. Because of the opposite effects of the step change and the predicted consumption innovation in 1979(1), we can not determine a priori the sign of the coefficient of d_{4t} . Equation (2.13) shows that the adjustment in 1971(1) has the expected sign. The size of the coefficient of d_{5t} on the contrary is different from the value predicted by the theoretical model. But as the estimate is highly insignificant we do not have to reject the theory. With respect to the size and the sign of the estimated parameters, the evaluation is rather tentative. Apart from the fact that we use the point estimates of θ , σ_v^2 and the relevant income

innovations, a reinterpretation of the formulae is needed, because we estimate the model from aggregate per capita data. As we have no data on the age structure of the population and distribution of labor income over different age groups at our disposal, we have chosen to adopt the procedure above followed.

From the empirical results we conclude that the life cycle model provides a rather good description of the data. Apart from the sign of the coefficient of d_{jt} , we find a confirmation of the theoretical model. A drawback seems to be the ad-hoc assumption of a structural change in the time preference parameter. In line with Hendry's (1979) criticism of ad-hoc modeling it seems worthwhile to try to revise the model in such a way that we do not have to appeal to this structural break. Given the empirical support of the life cycle model, we will look for a model that leaves the main features of the model intact. This strategy will be followed in the next section, where we show that a slightly revised model is capable to describe the consumption series, without calling on structural changes in parameters of consumer behavior.

3 The life cycle model under a moving planning horizon of constant length.

3.1 Theory.

In the life cycle model the consumer is assumed to be forward looking with a planning horizon that coincides with his expected lifetime. He distributes his life time wealth in an optimal way over present and future periods. He anticipates on possible income changes in the future in a rational way. An example is retirement. He also spreads the consequences of errors in forecasting income over the rest of his life, a feature which explains the great persistence of consumption (see e.g. Muellbauer (1983)). When a misinterpretation of the future actually takes place, the consumer will admit that he is not using his endowments in an optimal way and will replan his future consumption in the light of the new development. In replanning, it is of course not necessary to use a time horizon that equals the expected lifetime. When the consumer uses a shorter time horizon, forward looking behavior is still possible. It is not unrealistic to imagine that he will neglect periods far ahead in the future on which available information is scarce and unreliable, and will concentrate on more trustworthy information on the near future. Notice that it is possible that ex post the utility of the life time consumption under a moving planning horizon is greater than the satisfaction experienced by the "life cycle" adept.

In contrast to section 2, we assume that the consumer solves at each time period t the utility maximization problem

$$\begin{aligned} \text{MAX} \quad & \sum_{i=0}^T \beta^i U(c_{t+i}) \\ \text{S.T.} \quad & \sum_{i=0}^T (1+r)^{-i} c_{t+i} = (1+r)a_{t-1} + y_t + \sum_{i=1}^T (1+r)^{-i} E(y_{t+i} | I_t) \end{aligned} \quad (3.1)$$

where the length of the planning horizon T is postulated to shift along as time goes on. Solving the model for the utility function with constant absolute risk aversion (2.2) yields for the chosen consumption level

$$\eta_T c_t + \gamma^{-1} \ln[\beta(1+r)] r_T = (1+r) a_{t-1} + y_t + \sum_{i=1}^T (1+r)^{-i} E(y_{t+i} | I_t) \quad (3.2)$$

For the next period we find for c_{t+1}

$$\eta_T c_{t+1} + \gamma^{-1} \ln[\beta(1+r)] r_T = (1+r) a_t + y_{t+1} + \sum_{i=1}^T (1+r)^{-i} E(y_{t+1+i} | I_{t+1}) \quad (3.3)$$

Dividing by $1+r$, substituting $a_t = (1+r) a_{t-1} + y_t - c_t$ and subtracting (3.2) leads after some rearranging to

$$\begin{aligned} c_{t+1} - c_t &= \gamma^{-1} \ln[\beta(1+r)] - \eta_T^{-1} (1+r)^{-T} \gamma^{-1} (T+1) \ln[\beta(1+r)] \\ &\quad \eta_T^{-1} (1+r)^{-T} [E(y_{t+T+1} | I_t) - c_t] + \eta_T^{-1} [y_{t+1} - E(y_{t+1} | I_t)] \\ &\quad \eta_T^{-1} [\sum_{i=1}^T (1+r)^{-i} \{ E(y_{t+i+1} | I_{t+1}) - E(y_{t+i+1} | I_t) \}] \quad (3.4) \end{aligned}$$

Notice the great resemblance of (3.4) with expression (2.6). The main difference consists in the presence of some error correction term

$$\eta_T^{-1} (1+r)^{-T} [E(y_{t+T+1} | I_t) - c_t] \quad (3.5)$$

in (3.4). This term was found to yield favorable empirical results in Davidson et al (1978), where it was derived along completely different lines of reasoning. In our opinion, the introduction of a moving planning horizon provides an alternative explanation for the inclusion of an error correction mechanism in the consumption function. In the introduction we have stated that in this framework it is more appropriate to call it a correction term. The decision problem solved for period t yields for the "planned" consumption c_{t+1}^t

$$c_{t+1}^t = c_t + \gamma^{-1} \ln[\beta(1+r)] \quad (3.6)$$

From expression (3.4) follows

$$E(c_{t+1}|I_t) = c_t + \gamma^{-1} \ln[\beta(1+r)] - \eta_T^{-1}(1+r)^{-T} \gamma^{-1}(T+1) \ln[\beta(1+r)] \\ + \eta_T^{-1}(1+r)^{-T} [E(y_{t+T+1}|I_t) - c_t] \quad (3.7)$$

Comparing (3.6) and (3.7) shows that the "adjustment" can be expressed as

$$E(c_{t+1}|I_t) - c_{t+1}^t = -\eta_T^{-1}(1+r)^{-T} \gamma^{-1}(T+1) \ln[\beta(1+r)] + \\ \eta_T^{-1}(1+r)^{-T} [E(y_{t+T+1}|I_t) - c_t] \quad (3.8)$$

It is a result of the extra information on the future, which is taken into account by the consumer in period $t+1$. The terms "planned" and "adjustment" have been written between quotation-marks, provided the consumer knows that he will replan in the next period, he does not determine a planned consumption level for period $t+1$. When the consumer solves his maximization problem for the planning period, he will not make an error and this is the reason why we prefer to call $E(y_{t+T+1}|I_t) - c_t$ a correction term. In section 4 we will return to the consumption function derived by Davidson et al. In this section we are mainly concerned with the univariate process of consumption under a moving planning horizon, and we compare the resulting model with the one derived in subsection 2.1.

Subtracting the expression (3.4) for Δc_t from (3.4) yields

$$\Delta c_{t+1} = [1 - \eta_T^{-1}(1+r)^{-T}] \Delta c_t - \eta_T^{-1}(1+r)^{-T} \{E(y_{t+T+1}|I_t) - E(y_{t+T}|I_{t-1})\} \\ + \eta_T^{-1} \{y_{t+1} - E(y_{t+1}|I_t) + \sum_{i=1}^T (1+r)^{-i} [E(y_{t+1+i}|I_{t+1}) - E(y_{t+1+i}|I_t)]\} \\ - \eta_T^{-1} \{y_t - E(y_t|I_{t-1}) + \sum_{i=1}^T (1+r)^{-i} [E(y_{t+i}|I_t) - E(y_{t+i}|I_{t-1})]\} \quad (3.9)$$

In order to examine more deeply the dynamic properties of consumption, we assume that the change in income follows the stationary process (2.7). Using expression (2.8) and noting that

$$E(y_{t+T+1}|I_t) - E(y_{t+T}|I_{t-1}) = E(y_{t+T+1}|I_t) - E(y_{t+T+1}|I_{t-1}) \\ - E(y_{t+T}|I_{t-1}) + E(y_{t+T+1}|I_{t-1}) \quad (3.10)$$

$$= (\psi_0 + \dots + \psi_{T+1}) \nu_t + \delta + \sum_{j=1}^{\infty} \psi_{T+1+j} \nu_{t-j} ,$$

we find after substitution in (3.9)

$$\begin{aligned} \Delta c_{t+1} &= [1 - \eta_T^{-1} (1+r)^{-T}] \Delta c_t = \eta_T^{-1} (1+r)^{-T} \left\{ \delta + \sum_{j=1}^{\infty} \psi_{T+1+j} \nu_{t-j} \right\} + \\ &+ \eta_T^{-1} \left[\sum_{i=0}^T (\psi_0 + \dots + \psi_i) (1+r)^{-i} \right] \nu_{t+1} \\ &- \eta_T^{-1} \left[\sum_{i=0}^T (\psi_0 + \dots + \psi_i) (1+r)^{-i} - (\psi_0 + \dots + \psi_{T+1}) (1+r)^{-T} \right] \nu_t . \end{aligned} \quad (3.11)$$

Defining the consumption innovation ϵ_{t+1} as above $\epsilon_{t+1} = c_{t+1} - E(c_{t+1} | I_t)$, we have

$$\epsilon_{t+1} = \eta_T^{-1} \left[\sum_{i=0}^T (\psi_0 + \dots + \psi_i) (1+r)^{-i} \right] \nu_{t+1} , \quad (3.12)$$

From (3.11) it can be easily seen that when the change in income is generated by a MA-process of order q , the change in consumption follows an ARMA(1, max(1, q-T)) process. To derive the stochastic process for consumption when income is generated by an ARMA(p, q) model, it becomes necessary to explore the restrictions on the ψ 's implied by the p+q ARMA parameters. In Appendix C we show that in that case the change in consumption follows an ARMA(p+1, max(p+1, max(p-1, q) - T)) process.

When we compare the resulting models (2.9) and (3.11), we see that in the new situation, we have a different stochastic process for the change in consumption. Nevertheless, in both cases there is a 1-1 correspondence between the stochastic properties of income and consumption. Expressions (3.12) and (2.10) reveal that the consumption innovation is in very similar in both cases. The consequences of unanticipated structural changes in the ARMA parameters and/or the variance of the income process, for the variance of the consumption innovation are therefore the same. A major difference concerns the reaction to an unexpected structural change in the constant term of the income process. In the life cycle model of section 2.2 a shift

in the income drift gives rise to one step change in the constant for consumption. In the model with moving planning horizon we have besides the step change, a persistent adjustment of the constant term in the consumption process. We conclude also from expression (3.11) that the signs of the drift parameter in the consumption and income process coincide. It is especially this property that opens the possibility to drop the assumption of a structural change in the time preference parameter which we had to make in section 2.

Another advantage for estimation is the constancy of the innovation variance. When we assume that macro data per capita describe the behavior of a representative consumer, we may estimate model (3.11) with a constant variance. Notice that an adjustment of the parameters of time preference or risk aversion will lead to a step change in the consumption level for the model (3.11) too. The absence of these parameters in expression (3.9) results from the presumed constancy. The implications for the stochastic process of consumption can be traced by using expressions (3.2) and (3.3). It should be obvious that a change in β will not lead to a permanent change of the drift parameter of the model with moving time horizon.

3.2 Empirical results.

With income being generated by the process estimated in section 2.2, we find for the periods in which no structural change occurred an ARMA(1,1) model (3.13) for the change in consumption

$$(1 - \varphi_1 L) \Delta c_t = \eta_T^{-1} (1+r)^{-T} \delta + (1 - \theta_1 L) \varepsilon_t \quad (3.13)$$

with $\varphi_1 = 1 - \eta_T^{-1} (1+r)^{-T}$ and

$$\theta_1 = 1 - ((1-\theta)(1+r)^{-T} \eta_T^{-1} [1 - \theta + \theta \eta_T^{-1}]^{-1})$$

and θ being the MA-parameter of (2.12). It can easily be checked that the process satisfies the stability and invertibility conditions. As we have seen above a tentative investigation of the correlation structure for consumption suggested a random walk specification. The first question we have to answer is whether this empirical finding should lead to a rejection of the model (3.13). Defining θ_1^* such that $\theta_1 = \varphi_1 + \theta_1^*$, it is likely that

if θ_1^* is small compared with φ_1 , in small samples the ARMA(1,1) process is empirically equivalent to the random walk model as a result of the cancelling of the (almost) common root. We have calculated the values of the relevant parameters for a range of values of r and T , and the empirically found $\theta = .428$. In Table 3 we give some results for $r = .05$.

TABLE 3

T	φ_1	θ_1	θ_1^*	θ_1^*/φ_1	ρ_1	ρ_2	ρ_3
1	.512	.647	.135	.264	-.120	-.061	-.031
2	.683	.749	.066	.096	-.060	-.041	-.028
3	.768	.807	.039	.051	-.036	-.028	-.021
4	.819	.845	.026	.031	-.024	-.020	-.016
8	.909	.918	.008	.009	-.008	-.007	-.006

We see that with a planning horizon of 4 periods, θ_1^* covers only 3% of φ_1 , and with a time horizon of 8 periods the percentage has already fallen below 1%. In Table 3 we also report the theoretical values of the first three autocorrelations. As for large n (the number of observations) the sample autocorrelations are uncorrelated and normally distributed with standard deviations $n^{-1/2}$ (see Anderson (1971)), and the number of observations at our disposal is 71 ($1/\sqrt{71} = .119$), we conclude from the results of Table 3, that it is unlikely that the ACF is able to detect the ARMA(1,1) process. We conclude that the correlation structure has to be considered as not being incompatible with the theoretical model.

In section 3.1 we have argued that one of the consequences of a structural change in the constant term of the income process, is a step change in the consumption level. For the ARMA(1,1) process this leads to the introduction of two dummy variables. Using the appropriate expressions of the former section, it is straightforward to show that a change of δ to δ^* leads to a step change of the constant term in the ARMA model of size $(\delta^* - \delta)[1 + \eta_T^{-1}\tau_T]$ in the first period and of size $(\delta^* - \delta)[1 - \eta_T^{-1}\tau_T + \eta_T^{-1}(1+r)^{-T}(T+1)]$ in the next period. Therefore, as well in 1971 as in 1979 we expect a decrease of the constant term followed by an increase. We see here an alternative explanation for the clustering of outliers discussed in section 2.1. A correct interpretation of outliers, based on the economic theory, can also obviate the problem. The choice of an ARCH-model can in fact be

prompted by the incorrect handling of structural breaks.

The equation implied by the theory yields the following estimates

$$\begin{aligned} \Delta c_t = & .254\Delta c_{t-1} + 31.120d_{1t} - 89.500d_{2t} + 18.880d_{3t} + 25.120d_{4t} \\ & - 2.765d_{5t} - 8.149d_{6t} + 29.790d_{7t} + \epsilon_t - .538\epsilon_{t-1}, \\ \sigma^2(\epsilon_t) = & 564.0 \end{aligned} \quad (3.14)$$

where $d_{1t} = 1$ for 1967(2)-1971(1)

$d_{2t} = 1$ for 1971(1)

$d_{3t} = 1$ for 1971(2)-1979(1)

$d_{4t} = 1$ for 1971(2)

$d_{5t} = 1$ for 1979(1)

$d_{6t} = 1$ for 1979(2)-1984(4)

$d_{7t} = 1$ for 1979(2).

As our computer programs do not enable us to obtain ML-estimates for models like (3.11), we have used an algorithm proposed by Spliid (1983). This method yields moment estimators which are asymptotically normally distributed and strongly consistent. For details we refer to Spliid (1983).

The residuals do not exhibit much autocorrelation. For the ACF only r_{16} has a significant value and the PACF has no significant values. We have four outliers in 1976(1), 1978(1), 1981(1) and 1982(2). The values of the BP and LB test statistic based on the first 4, 8, 12 and 16 residual autocorrelations are reported in Table 4. They are highly insignificant. According to the test statistics stationarity and normality of ϵ_t do not have to be rejected. These empirical results are in accordance with the theoretical model.

TABLE 4 Test statistics for model (3.14)

p	BP	LB
4	.32	.33
8	2.37	2.48
12	6.28	6.55
16	10.98	11.45
$\eta(1)$	1.07	
$\eta(4)$	1.71	
s_1	-.10	
s_2	-.07	

Let us now examine the sign and size of the parameter estimates. First we consider the value of the consumption variance. Using expression (3.12) we have in this instance

$$\varepsilon_t = (1 - \theta + \theta \eta_T^{-1}) \nu_t, \quad (3.15)$$

where θ denotes the MA parameter of the income process. With the positive value of θ , the variance of the income innovation should exceed that of the consumption innovation. The values reported in (3.14) and (2.12) are in agreement with the theory. Next we consider the values of φ_1 and θ_1 . Theoretically they should be positive and smaller than 1, a criterion which is satisfied. From the values reported in Table 3, we infer that the point estimates of the AR and MA parameters are rather small. Notice however that the estimated standard errors of .342 and .310 respectively prevent us from drawing sharp conclusions. Unfortunately, the low value of the AR parameter is prohibitive for giving an indication of the length of the planning horizon.

In section 3.1, we have shown that the sign of the constant term of the process for consumption should be the same as that for the income process. From expression (3.13) it follows that the (absolute value of the) constant term of the income process exceeds that of consumption. A comparison of (3.14) with (2.12) shows this requirement is satisfied. The ratio of the constant terms equals in all cases $\eta_T^{-1}(1+r)^{-T}$. Empirically we find the

values .769, .749 and .626 respectively. The AR parameter equals $1 - \eta_T^{-1}(1+r)^{-T}$. This yields an extra point estimate .746 of $\eta_T^{-1}(1+r)^{-T}$. These values support the statement that r and/or T have not undergone a structural shift. A test of the equality of the ratios, can only be performed when the joint (singular) process for consumption and income has been estimated.

To appraise the coefficients of the dummy variables, remember that they absorb the joint effect of the step change and the transformed income innovation. Using the estimates of the income innovation, the constant term and the MA parameter of (2.12), we may infer a negative sign for d_{2t} and a positive one for d_{7t} . Because of the size of the income innovation in 1971(2) in relation to the step change, we may expect a positive coefficient for d_{4t} . The sign of the coefficient of d_{5t} is unpredictable. Equation (3.14) shows that all the empirical results are in accordance with the theoretical implications. A warning is however appropriate, because of the insignificance of all the coefficients except that of d_{2t} . Moreover, in applied work it is difficult to pinpoint the exact moment of appearance of the structural change. This problem is probably inherent in empirical econometrics. Notwithstanding all the limitations, it should be obvious that the theoretical framework provides a basis for interpreting outliers. From the empirical analysis of this section we conclude that the model for the forward looking consumer with a moving planning horizon of constant length provides a good description of the consumption series. The model does not rely on an ad-hoc assumption about structural changes and removes in this way an important disadvantage of the life cycle model.

4. Relationships with existing models.

This section is devoted to a discussion of the relationships between the models put forward in this paper and two models from the literature. In the first subsection we compare the life cycle model of section 2.1 with Hall's (1978) model. The second subsection is devoted to a more comprehensive discussion of the model with a moving planning horizon. In particular we show that if the change in income is generated by an autoregressive process of order 1, equation (3.4) is highly similar to the model of Davidson et al. (1978).

4.1 Hall's model.

In Hall's famous article on consumption, the consumer is assumed to maximize at each period t the expected present value of the utility of life time consumption subject to the budget constraint

$$\begin{aligned} \text{MAX } E \left(\sum_{i=0}^{T-t} \beta^i U(c_{t+i}) \mid I_t \right) \\ \text{S.T. } \sum_{i=0}^{T-t} (1+r)^{-i} c_{t+i} = (1+r)a_{t-1} + \sum_{i=0}^{T-t} (1+r)^{-i} y_{t+i} \end{aligned} \quad (4.1)$$

Hall shows that the first order conditions for (4.1)

$$E(U'(c_{t+i}) \mid I_t) = [\beta(1+r)]^{-i} U'(c_t) \quad , i = 1, \dots, T-t \quad (4.2)$$

have implications for the time series properties of consumption. In line with Hall's work, we have assumed in section 2.1 that the only source of uncertainty concerns future labor income. Comparing (2.1) with (4.1) shows the great resemblance and reveals immediately that the difference consists in the amount of information on the stochastic process of income that is used by the consumer. In the models of sections 2 and 3, only the first (conditional) moments of the income process are needed to solve the

intertemporal optimization problem, whereas the more sophisticated consumer studied by Hall incorporates in principle all the stochastic information on income. Notice however that a specific choice of the utility function possibly restricts the amount of information, that is required for intertemporal utility maximization. For a quadratic utility function the models are equivalent. Palm and Winder (1987) investigate the model (4.1) for the utility function with constant absolute risk aversion under the additional assumption of normality of consumption. A sufficient condition for this assumption is normality of income.

For $i=1$ expression (4.2)

$$E(\exp(-\gamma c_{t+1}) | I_t) = [\beta(1+r)]^{-1} \exp(-\gamma c_t) \quad (4.3)$$

can be rewritten in that case as

$$E(c_{t+1} | I_t) - c_t = \gamma^{-1} \ln[\beta(1+r)] + 1/2 \gamma V(c_{t+1} | I_t) \quad (4.4)$$

where $V(c_{t+1} | I_t)$ denotes the conditional variance. The consumption innovation $\varepsilon_{t+1} = c_{t+1} - E(c_{t+1} | I_t)$ is given by

$$\varepsilon_{t+1} = \eta_{T-t-1}^{-1} \{ y_{t+1} - E(y_{t+1} | I_t) + \sum_{i=2}^{T-t} (1+r)^{-i} [E(y_{t+i} | I_{t+1}) - E(y_{t+i} | I_t)] \} \quad (4.5)$$

Notice that this expression is identical to the one for the model described in section 2.1. As $V(c_{t+1} | I_t) = V(\varepsilon_{t+1})$, we see that with a homoscedastic income process both models are observationally equivalent. The two specifications are empirically distinguishable from each other when an unexpected change in the variance of the income process arises. This change will only affect the variance of the disturbance term in (2.6), but in the model (4.4) for the more sophisticated consumer it will also have a persistent effect on the drift parameter. There is an interesting possibility of relaxing the postulate of normality in favor of the assumption of conditional normality of the ARCH type. In that case we can discriminate between the two specifications. This example illustrates that Hall's conclusion that all the relevant information of the past is incorporated in c_t , is not necessarily correct. It also illustrates that

in general a different assumption about the stochastic behavior leads to a different operational model.

We have investigated the model (4.1) with moving planning horizon too. For the utility function with constant absolute risk aversion combined with normality, it can be shown that in case of a homoscedastic income process the models (2.1) and (4.1) are observationally equivalent. In conclusion, the empirical results of sections 2.2 and 3.2 remain valid in the more comprehensive framework introduced in this section.

4.2 The Davidson, Hendry, Srba and Yeo model.

To illustrate the concept of the correction term, we will show that if the change in income follows an autoregressive process of order 1, the model presented in section 3.1 yields a relationship between consumption and income that is similar to the mechanism put forward by Davidson et al (1978). In this section we assume that

$$\Delta y_t = \varphi^* \Delta y_{t-1} + \nu_t \quad (4.6)$$

For the sake of simplicity we have omitted the constant term, of which the incorporation does not change the conclusions. The expression of interest is (3.4). To calculate the relevant conditional expectations we call on the formulae (2.7) and (2.8) with

$$\psi_i = \varphi^{*i} \quad \text{for all } i.$$

It is however more convenient to calculate the conditional expectations directly as follows

$$y_{t+1} = E(y_{t+1} | I_t) = \Delta y_{t+1} = \varphi^* \Delta y_t \quad (4.7)$$

$$E(y_{t+1}|I_{t+1}) - E(y_{t+1}|I_t) = \sum_{j=0}^{i-1} \varphi^{*j} [y_{t+1} - E(y_{t+1}|I_t)] \quad (4.8)$$

$$E(y_{t+T+1}|I_t) = y_t + \left[\sum_{j=1}^{T+1} \varphi^{*j} \right] \Delta y_t \quad (4.9)$$

Substituting (4.7), (4.8) and (4.9) into expression (3.4) gives after some rearranging

$$\Delta c_{t+1} = \alpha_0 + (\alpha_1 - \alpha_2) \Delta y_{t+1} + \alpha_2 \Delta \Delta y_{t+1} + \alpha_3 (y_t - c_t) \quad (4.10)$$

with

$$\alpha_0 = \gamma^{-1} \ln[\beta(1+r)] \{1 - \eta_T^{-1} (1+r)^{-T(T+1)}\}$$

$$\alpha_1 = \eta_T^{-1} \left[\sum_{i=0}^T (1+r)^{-i} (1+\varphi^* + \dots + \varphi^{*i}) \right]$$

$$\alpha_2 = \eta_T^{-1} \left[\sum_{i=0}^{T-1} (1+r)^{-i} (\varphi^* + \dots + \varphi^{*i+1}) \right]$$

$$\alpha_3 = \eta_T^{-1} (1+r)^{-T}$$

Expression (4.10) shows that we have the same mechanism as found in Davidson et al. In each time period consumers spend the same as they spent the period before, modified by a proportion of the change and the change of the change in income, and by a term labeled and interpreted by Davidson et al. as the error correction term. With our economic model we can determine the sign and size of the coefficients. For α_3 we find that it should be positive and smaller than 1. The sign and size of both $\alpha_1 - \alpha_2$ and α_2 depend on the sign of φ^* . It is easy to show that if $0 < \varphi^* < 1$ we have $0 < \alpha_1 - \alpha_2 < 1$ and $0 < \alpha_2$, and when $-1 < \varphi^* < 0$: $1 < \alpha_1 - \alpha_2$ and $\alpha_2 < 0$. In their empirical analysis, Davidson et al. have found a coefficient for the change of income between 0 and 1, and a negative coefficient for the change of the change in income. Unfortunately, we are not allowed to refer to their finding as an empirical confirmation of our model. Notice, that Houthakker and Taylor (1970) find in their model with habit formation a positive influence for $\Delta \Delta y_t$ on c_t . An advantage of the model with moving planning horizon is that it meets the

objection raised in many contributions on error correction mechanisms (see e.g. Salmon(1982), Klok(1984), Currie(1981)) that the interpretation of the correction term as an error correction term encounters difficulties in case of a "linear trending target", to use Klok's terminology.

There remain important differences between specification (4.10) and the consumption function found by Davidson et al. The main differences relate to the role of the inflation variable and the log-linear functional form. We have the impression that with respect to the functional form, a possible solution might be the use of the utility function with constant relative risk aversion. Palm and Winder (1987) have investigated this utility function and the analysis led to a loglinear specification for the consumption model. A nice feature of this utility function is the property of homotheticity, which allows to establish a connection with the original papers of the Life Cycle Hypothesis.

5 Summary and conclusions.

In this paper we analyzed forward looking consumption behavior where the consumer replans his consumption and his savings each period. Special attention was devoted to the consequences of structural changes in the income process, which because of replanning, will have an impact on the decision procedure. These consequences yield extra opportunities to test the theoretical model. We stressed the similarity of the life cycle models with the setup in controlled experiments. The important difference however concerns the lack of experimental data. This results for instance in difficulties to determine the moment and the nature of the structural shifts.

In the first stage we analyzed the life cycle model, and we gave an economic argument for the plausibility of the appearance of ARCH processes. However, we found no empirical evidence for existence of these structures. A possible explanation could be aggregation: we have estimated and tested the model with macro data per capita. On the whole the empirical results are quite favorable for the life cycle hypothesis, apart from the fact that we were forced to assume a structural change in one of the parameters that characterize consumption behavior.

An alternative which does not rely on the ad-hoc assumption of a structural change in the time preference parameter, is the model in which the consumer uses a moving planning horizon of constant length. The implied relationship between consumption and income incorporates a correction term, which has interesting properties (see e.g. Davidson et al (1978)). Our framework provides an new, economic explanation for the appearance of an error correction mechanism. The analysis of the consequences of unanticipated structural changes in the income series, revealed clusters of outliers. In section 2 this was used as an argument for assuming an ARCH-structure for the disturbances. Here a different aspect is illuminated. When a test for ARCH structures takes a significant value because of a cluster of outliers, it is very likely that this is a result of the improper handling of structural changes. From above we may conclude that the framework of intertemporal maximization is a suitable one for the interpretation of outliers. The empirical analysis of the model showed

that this framework can provide a satisfactory description of the data, without having the disadvantage inherent in the life cycle model.

Throughout the paper we made the assumption of a constant real interest rate. A possible extension deals with variable interest rates. For the life cycle model this can be done without too many problems along the lines of Hansen and Singleton (1982, 1983) and Palm and Winder (1987). In the model with moving planning horizon, this extension is however far from being straightforward. Fortunately, an analysis of the consumption series that is compatible with forward looking behavior and that accounts for the serial correlation of the series in the "roaring seventies" and "early eighties" is possible as we have tried to show in this paper.

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Appendix A Sources of the data.

The quarterly series on nondurable consumption per capita in prices of 1980 has been computed as the sum of consumption expenditures per capita on food and beverages and services and other nondurables. Monthly indices on these series and on total consumption expenditures are published in Centraal Bureau voor de Statistiek, Maandstatistiek Binnenlandse Handel en Dienstverlening, Staatsuitgeverij, 's Gravenhage. Annual figures on expenditures which are published in Centraal Bureau voor de Statistiek, Nationale Rekeningen, Staatsuitgeverij, 's Gravenhage, have been used to transform the indices into monthly expenditures per capita expressed in prices of 1980. The monthly figures have then been aggregated into quarterly data. To remove the seasonal pattern in the ratio of nondurable and total consumption, we have calculated the nondurable consumption shares as a moving average of the ratios.

The income and consumption series in figures 1 and 2 have been obtained from the Centraal Plan Bureau.

Appendix B Empirical results for real nondurable consumption per capita.

The expressions and tables in this appendix are denoted by a number combined with a "'-sign. The numbers correspond to those used in the text for real total consumption per capita. With respect to the evaluation of the size and sign of the parameter estimates we refer to the discussion of the empirical results for total consumption in sections 2.2 and 3.2. Here we want to confine ourselves to the conclusion that the results for real nondurable consumption per capita are grosso modo the same as those for real total consumption per capita.

$$\begin{aligned} \Delta c_t &= 20.60d_{1t} - 5.85d_{2t} - 49.66d_{3t} + 11.71d_{4t} - .57d_{5t} & (2.13)' \\ &\quad (6.67)^{1t} \quad (1.22)^{2t} \quad (2.30)^{3t} \quad (.54)^{4t} \quad (.03)^{5t} \\ \sigma^2(\epsilon_t) &= 457.78 \end{aligned}$$

The model with two separate slope coefficients α_1 and α_2 for the subperiods 1967(2)-1970(4) and 1971(1)-1979(4) yields estimates $\hat{\alpha}_1=26.38$ (4.80) and $\hat{\alpha}_2=17.97$ (4.85). A t-test of the equality of α_1 and α_2 has an insignificant value : $t(65)=1.268$.

Table 2' Test statistics for model (2.13)'

p	BP	LB
4	2.63	2.74
8	3.91	4.20
12	7.29	8.30
16	15.76	19.49
$\eta(1)$.18	
$\eta(4)$.99	
S_1	-.18	
S_2	.09	

$$\Delta c_t = \frac{.221\Delta c_{t-1}}{(.755)} + \frac{25.070d_{1t}}{(2.589)} - \frac{60.590d_{2t}}{(3.237)} + \frac{13.040d_{3t}}{(2.436)} + \frac{35.670d_{4t}}{(1.564)} + \frac{13.030d_{5t}}{(.707)} - \frac{3.031d_{6t}}{(1.472)} + \frac{20.760d_{7t}}{(.838)} + \varepsilon_t - \frac{.550\varepsilon_{t-1}}{(2.067)} \quad (3.14)'$$

$$\sigma^2(\varepsilon_t) = 327.0$$

Table 4' Test statistics for model (3.14)'

p	BP	LB
4	.32	.33
8	.86	.90
12	6.41	6.69
16	9.97	10.40
$\eta(1)$.02	
$\eta(4)$	4.96	
S_1	-.15	
S_2	-.03	

Appendix C The stochastic process for consumption in the model with moving planning horizon.

We consider a stationary invertible ARIMA(p,1,q) process for income

$$\Phi(L)\Delta y_t = \Theta(L)\nu_t \quad , \text{with } E\nu_t = 0 \text{ and } \sigma^2(\nu_t) = \sigma^2 \quad . \quad (C.1)$$

The lag polynomials are defined as

$$\Phi(L) = \varphi_0 - \varphi_1 L - \dots - \varphi_p L^p \quad , \varphi_0 = 1 \quad (C.2)$$

$$\text{and } \Theta(L) = \theta_0 - \theta_1 L - \dots - \theta_q L^q \quad , \theta_0 = 1 \quad (C.3)$$

respectively. The MA(∞)-representation is denoted as

$$\Delta y_t = \Psi(L)\nu_t \quad (C.4)$$

with

$$\Psi(L) = \sum_{i=0}^{\infty} \psi_i L^i \quad , \psi_0 = 1 \quad . \quad (C.5)$$

From (C.1) and (C.4) follows

$$\Phi(L)\Psi(L) = \Theta(L) \quad . \quad (C.6)$$

This can be used to trace the restrictions on the parameters ψ_i , implied by the p+q ARMA parameters. It is straightforward to show that (C.6) implies

$$\psi_j = \varphi_1 \psi_{j-1} + \dots + \varphi_p \psi_{j-p} \quad \text{for all } j \geq \max(p, q+1) \quad . \quad (C.7)$$

Therefore, for $j \geq \max(p, q+1)$ the parameters ψ_j are generated by a p^{th} order homogeneous difference equation. When we define μ_t as the MA part of

(3.11), we have

$$\begin{aligned} \mu_t &= \eta_T^{-1} \left[\sum_{i=0}^T (1+r)^{-i} (\psi_0 + \dots + \psi_i) \right] \nu_t \\ &\quad - \eta_T^{-1} \left[\sum_{i=0}^T (1+r)^{-i} (\psi_0 + \dots + \psi_i) - (1+r)^{-T} \sum_{i=0}^{T+1} \psi_i \right] \nu_{t-1} \\ &\quad + \eta_T^{-1} (1+r)^{-T} \sum_{j=2}^{\infty} \psi_{T+j} \nu_{t-j} \end{aligned} \quad (C.8)$$

Calculating the autocovariance function for μ_t yields for all $i \geq 2$

$$\begin{aligned} E(\mu_t \mu_{t-i}) &= \sigma^2 \left\{ \psi_{T+i} (1+r)^{-T} \eta_T^{-2} \left[\sum_{j=0}^T (1+r)^{-j} (\psi_0 + \dots + \psi_j) \right] \right. \\ &\quad - \psi_{T+i+1} (1+r)^{-T} \eta_T^{-2} \left[\sum_{j=0}^T (1+r)^{-j} (\psi_0 + \dots + \psi_j) - (1+r)^{-T} \sum_{j=0}^{T+1} \psi_j \right] \\ &\quad \left. + \eta_T^{-2} (1+r)^{-2T} \sum_{j=2}^{\infty} \psi_{T+i+j} \psi_{T+j} \right\} \end{aligned} \quad (C.9)$$

For every i satisfying $T+i \geq \max(p, q+1)$ we can use (C.7) to rewrite (C.9) as

$$\begin{aligned} E(\mu_t \mu_{t-i}) &= \sigma^2 \sum_{j=1}^p \varphi_j \left\{ \psi_{T+i-j} (1+r)^{-T} \eta_T^{-2} \left[\sum_{k=0}^T (1+r)^{-k} (\psi_0 + \dots + \psi_k) \right] \right. \\ &\quad - \psi_{T+i-j+1} (1+r)^{-T} \eta_T^{-2} \left[\sum_{k=0}^T (1+r)^{-k} (\psi_0 + \dots + \psi_k) - (1+r)^{-T} \sum_{k=0}^{T+1} \psi_k \right] \\ &\quad \left. + \eta_T^{-2} (1+r)^{-2T} \sum_{k=2}^{\infty} \psi_{T+i-j+k} \psi_{T+k} \right\} \end{aligned} \quad (C.10)$$

For every i satisfying in addition $T+i-p \geq T+2$, substitution of (C.9) in (C.10) gives as a result

$$E(\mu_t \mu_{t-i}) = \varphi_1 E(\mu_t \mu_{t-(i-1)}) + \dots + \varphi_p E(\mu_t \mu_{t-(i-p)}) \quad (C.11)$$

From the requirements in (C.9), (C.10) and (C.11), we see that for all $i \geq \max(p+2, \max(p,q+1)-T)$ the autocovariances of μ_t are generated by a p^{th} order homogeneous difference equation. For an ARMA(r,s) process the autocovariances γ_j are generated for all $j \geq s+1$ by a r^{th} order homogeneous difference equation (see Box and Jenkins (1970)). Because the autocovariance function determines the order of a stationary stochastic process, we conclude from (C.11) that μ_t follows an ARMA(p, max(p+1, max(p-1,q)-T)) process, where the AR part coincides with the one for the income process. Say

$$\Phi(L)\mu_t = \tilde{\Theta}(L)\zeta_t, \quad E\zeta_t = 0 \text{ and } \sigma^2(\zeta_t) = \sigma_\zeta^2 \quad (C.12)$$

or in the MA-representation

$$\mu_t = \Phi^{-1}(L)\tilde{\Theta}(L)\zeta_t \quad (C.13)$$

Substituting (C.13) in (3.7) leads to the conclusion that c_t is generated by an ARIMA(p+1, 1, max(p+1, max(p-1,q)-T)) process, with the autoregressive part including that of the income process.